2203000205023004
EXAMINATION FEBRUARY -MARCH 2024
BACHELOR OF SCIENCE (FIFTH SEMESTER)
MATHEMATICS-XIV
(MTH-504-REAL ANALYSIS-II) LEVEL-2

[Time: As Per Schedule] **[Max. Marks:50]**

Instructions:
1. Fill up strictly the following details on your answer book
   a. Name of the Examination: **BACHELOR OF SCIENCE (FIFTH SEMESTER)**
   b. Name of the Subject: **MATHEMATICS-XIV (MTH-504-REAL ANALYSIS-II) LEVEL-2**
   c. Subject Code No: **2203000205023004**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow **usual** notations.

Seat No:

Student's Signature

Q.1 **Answer any FIVE as directed.** **10**

1) If the real-valued function f is continuous at $a \in \mathbb{R}^{\mathbf{1}}$, then prove that $|f|$ is also continuous at $a \in \mathbb{R}^{\mathbf{1}}$.
2) If $|x-2| < 1$, then prove that $|x^2 + 2x - 8| < 7$.
3) If f is a real valued function, then define $\lim_{x\to\infty} f(x) = L$.
4) Define convergent sequence in a Metric Space with an illustration.
5) State the usual metric for $\mathbb{R}$ and the discrete metric for $\mathbb{R}$.
6) Show that any subset of R_d is open.
7) Show that every Cauchy sequence in R_d is convergent.
8) Give an example of a subset B of a set A in a metric space $\langle M, \rho \rangle$, such that B is open in A, but B is not open in M.

Q.2 Attempt any TWO. **10**

1) Let $x = \langle x1, x2, \ldots, xn\rangle$ and $y = (y1, y2, \ldots, y\text{n})$ of $\mathbb{R}^n$.
   If $d(x, y) = \sum_{i=1}^{n} |x_i - y_i|$, then show that $\langle \mathbb{R}^n, d\rangle$ is a metric space.

2) If ρ and σ are metrics for set M , then prove that $\rho + \sigma$ is also metric for M.

3) If the real valued function f and g are continuous at a $\in \mathbb{R}^1$, then prove that $f \cdot g$ and $max\{f, g\}$ are also continuous at a $\in \mathbb{R}^1$.

Q.3 Attempt any TWO. **10**

1) Show that any convergent sequence in a metric space $\langle M, \rho\rangle$ cannot converge to two distinct points of M.

2) Let $\langle M, \rho\rangle$ and $\langle M, \sigma\rangle$ be two metric spaces , if there exists $k > 1$ such that $\frac{1}{k}\ \sigma\langle x, y\rangle \leq \rho\langle x, y\rangle \leq k\langle x, y\rangle; x, y \in M$, then prove that $\langle M, \rho\rangle$ and $\langle M, \sigma\rangle$ are equivalent.

3) For each $n \in I$, let $P_n = \langle x_n, y_n\rangle$be a point in $\mathbb{R}^2$. Show that a sequence $\{P_n\}_{n=1}^{\infty}$ converges to $P = \langle x, y\rangle$ in $\mathbb{R}^2$ if and only if $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ converge in $\mathbb{R}^1$ to x and y respectively.

Q.4 Attempt any TWO. **10**

1) Prove that the real valued function f is continuous at $a \in \mathbb{R}^1$ if and only if given $\epsilon > 0, \exists \delta > 0$, such that $f^{-1}(B[\,f(a); \epsilon]) \supset B[a; \delta]$.

2) Prove that the function f is continuous at $a \in M_1$ if and only if whenever $\{x_n\}_{n=1}^{\infty}$ is sequence of points in M_1 converging to a, then the sequence $\{f(x_n)\}_{n=1}^{\infty}$ of points converges to $f\ (a) \in M_2$.

3) Prove that the if both real value function f and g are continuous at a point a, then prove that $max\{f, g\}$ and $min\{f, g\}$ are also continuous at a.

Q.5 Attempt any TWO. **10**

1) Let $\langle M_1, \rho_1\rangle$ and $\langle M_2, \rho_2\rangle$ be two metric spaces let $f : M_1 \to M_2$. If $f^{-1}(G)$ is open in M_1 whenever G is open in M_2, then show that the f is continuous on M_1.

2) Let $\langle M, \rho\rangle$ be a metric space. Let A be a proper subset of M, then prove that the subset G_A of A is an open subset of metric space $\langle A, \rho\rangle$ if and only if there exists an open subset G_M of $\langle M, \rho\rangle$, such that $G_A = A \cap G_M$.

3) Define open set in a metric space. Prove that an open interval (a, b) in $\mathbb{R}$ is open set in $\mathbb{R}$.

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